# DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TEXAS AT AUSTIN ADVICE FOR STUDENTS WRITING REPORTS, THESES AND DISSERTATIONS

Written by Ward Cheney, incorporating suggestions from many sources. Version of May 24, 2000

The writing of a report, thesis, or dissertation is a serious undertaking, and one of its purposes is purely pedagogical. Namely, the candidate is expected to learn how to write in a clear expository style about mathematical matters. The writing is quite separate from the study and research that go into the work prior to writing. It demands rather a different set of skills, and these do not come easily to everyone. Besides basic English writing skills, a knowledge of accepted mathematical style is also required.

Among all human endeavors, mathematics is preëminent in its striving for absolute precision in its formal written text. Precision in writing is not easily attained, but one begins by using always the *correct* word at the *proper place* and by carefully *constructing* each sentence. We also advise against the use of slang, colloquialisms, and other non-standard linguistic devices. (More about that later.)

Let us assume that your materials have been assembled in the form of notes. You should next prepare an outline of your entire project. There should be a number of chapters, starting with an introductory one from which a reader can learn what you intend to do in the later chapters. Your outline should be in detail, enumerating all lemmas, definitions, and theorems if you are writing a dissertation. You should let your advisor see this outline before you do much writing.

After you have done a small amount of the writing (following your outline, of course), you should let your advisor see that. Perhaps he/she will have some "global" suggestions that will apply to everything you write. There may be matters of style that need to be brought to your attention.

After your advisor has read your text (all or part) and you have revised it (taking account of his/her suggestions), give back to him/her not only your new text but also the copy of the previous version in which the suggestions were indicated. This is very important, as it may spare the advisor the time and effort of re-reading portions of the text that were already satisfactory. Also, she/he will want to see how and whether you have dealt with the previous suggestions.

Always proof-read your own text carefully before giving it to your advisor to read. He/she should not have to serve as a proof-reader. You are responsible for accuracy of spelling, proper punctuation, good grammar, adherence to recognized mathematical style, and mathematical correctness. Of course, your advisor will help you to eliminate errors, but you should catch most of your own.

If English is not your native language, you may need extra help with sentence structure and other details. (If English is your native language, you may still require help with those matters!)

All text must be prepared eventually in TeX. In this day of modern mathematical typesetting methods, no other system makes much sense. You can do the TeX-ing yourself or pay to have it done. In any case, the results will be quite pleasing to the eye, and you will be very proud of your work. It is strongly recommended that you do your own TeX-ing. A knowledge of TeX is a great advantage for your future mathematical career or for any other career in which the writing of technical reports will be necessary. Be sure to allow plenty of time for learning the TeX system. Our department can provide a computer account for you to use in preparing your text in TeX. Departmental SUNs run the UNIX operating system, and you will need to learn that too. Knowledge of the UNIX system is another useful addition to your skills and abilities. Apply for a computer account in the departmental office.

The sooner you learn TEX the better! You can begin your dissertation or thesis NOW in TEX. For example, you can certainly start your bibliography or list of references. Get the form right at the beginning, and add gradually to the list. Don't worry about having too many references. Any that you do not want to print in the final dissertation can be prefaced with the symbol % in your computer file. It is not too early to acquire the files needed for the special UT Dissertation format.

Many mathematicians prefer La-T<sub>E</sub>X to T<sub>E</sub>X. It has special provisions for keeping track of equation numbers and theorem numbers. Also it has an automatic bibliographic system. Our professional T<sub>E</sub>X expert, Ms. Margaret Combs, prefers "plain" T<sub>E</sub>X, however, for its additional versatility.

If you want your advisor to examine a manuscript (i.e., text written by hand), it should be as close as possible to your intended typescript. In other words, advisors do not want to look at rough notes.

## Specific Style Advisories

Here are some important matters of style that good mathematical writers try to observe meticulously.

- 1. Sentences should NOT begin with mathematical symbols. For example, do not say " $\psi$  is a wavelet." Say "The function  $\psi$  is a wavelet".
- 2. Symbols should not appear in titles of dissertations, reports, chapters and sections. Symbols in titles are a bibliographer's

nightmare; they are often simply omitted in bibliographies because the symbol fonts are not available. Symbols in titles are also very un-inviting. Do you think you would like to read a paper whose title is "Applications of the PVZ theorem to spaces  $A_{\phi}^{\lambda}(\Omega_q)$ "? One can argue that if the title makes no sense to you, the paper can be ignored. But that would not be an informed decision. Since the question of titles has arisen, this is an opportunity to caution against titles that are meaningless. The classic example of the latter is "On a Theorem of Erdős". (The accent " is rendered in TEX by \H followed by a space. It is not the same as ".)

- 3. Use English descriptions and English text in preference to mathematical symbolism wherever possible. For example, you can say "Let f be a continuous function on S" rather than "Let  $f \in C(S)$ ." (That is just a minor example.) One reason for this advice is that it makes for smoother reading, and another is that the meanings of symbols often change in a rather short time, whereas the meanings of words change very slowly. Furthermore, you cannot expect every reader to be familiar with all your notation, some of which may be non-standard. Finally, mathematical symbolism is by its nature INTIMIDATING, even to mathematicians. There is nothing so daunting as having to read a page of formulas! Keep the formulas to a minimum and avoid symbols if ordinary language will do as well. There may be cases where, for good reason, one wishes to violate this rule. But it should be a good reason!
- 4. Don't use one letter to denote two different mathematical objects in the same proof. In fact, it is better not to use the same letter to signify different things within an entire chapter. Do not use  $\phi$  and  $\varphi$  in the same discussion. It is too demanding of the reader to maintain this fine distinction. On the other hand, you should use  $\in$  for membership in a class, and  $\epsilon$  for the variable. So these two similar symbols may appear near each other without confusing the reader. Other pairs of similar letters that you should try to avoid in close proximity are  $(a, \alpha)$ ,  $(v, \nu)$ ,  $(n, \eta)$ ,  $(k, \kappa)$ ,  $(u, \mu)$ ,  $(p, \rho)$ ,  $(t, \tau)$ ,  $(x, \chi)$ , and  $(w, \omega)$ .
- 5. Eschew these monstrosities in formal writing: "s.t." (for "such that"), ∀, ∃, ⇒, ⇒, ↔, WLOG (meaning "without loss of generality"—one can argue that it may also stand for "WITH loss of generality".) This sort of notation saves time in the discovery or inventive phase of a mathematician's work, and one cannot challenge its usefulness in that context. Most of it should be boldly excised in the formal writing. For example, why say "If B is an operator such that A ↔ B..." when one can say "If B is an operator that commutes with A..."? (This example is from a famous book.) Two advantages accrue from the simple English wording: first, it is felicitous and non-intimidating. Second, readers who do not know the symbol still will undoubtedly know about things that commute with each other. One remark about "s.t.": among optimization specialists it means "subject to", while others understand it to mean "such that". A remark about ∋: among logicians it has been used for "such that", while others use it in the form A ∋ x to mean x ∈ A. Ambiguity plagues these special symbols! Use them not at all or sparingly and with good reason.
- 6. Do not confuse f (a function) with f(x) (one of its values). There are alternative notations especially designed to help in this situation, namely, the notations illustrated by  $x \mapsto \cos x$  and  $\cos(\bullet)$ , for example.
- 7. Don't split infinitives. Here is an example of a split infinitive: "He tried to carefully prove his theorem." Here are acceptable versions: "He tried to prove his theorem carefully"; "He carefully tried to prove his theorem"; "He tried carefully to prove his theorem"; "He tried to prove carefully his theorem". So— out of the five versions given, do not perversely choose the one that is wrong.
- 8. Use "which" and "that" correctly; they are not interchangeable. Here are some correct examples: "Let f be a function that has two derivatives." "The Fourier transform, which can be defined on  $L_1(-\infty,\infty)$ , is a linear mapping." It must be admitted that some authorities do not insist on this distinction, but others do, and since mathematical writing should epitomize precise use of words, we can take advantage of the distinction between the two words. "That" is used for a restrictive clause, and "which" for a non-restrictive clause. Fowler (see the reference list) has said "...if writers would agree to regard 'that' as the defining relative pronoun, and 'which' as the non-defining, there would be much gain in both lucidity and in ease." In all honesty, I should tell you that he goes on to say "Some there are who follow this principle now; but it would be idle to pretend that it is the practice either of most or of the best writers."
- 9. Avoid this sort of construction: "Let  $x \in \mathbb{R}$  be a point...". The trouble here is that " $\in$ " means "belongs to", and the resulting clause has two verbs (one too many). Here, item 3 above is helpful: simply write "Let x be a point of  $\mathbb{R}$  ...". For the same reasons, this is bad: "Let  $G \subset \mathbb{C}$  be a domain such that...". You can say instead, "Let G be a domain in  $\mathbb{C}$  such that...." Don't say "If  $f \in H$  then  $g(x) = \sum a_i f(x+i)$  defines a function g..." Here again, the equal sign serves as a verb, and then one sees another verb "defines" in the same clause, so it doesn't read correctly. Also, the reader thinks that  $f \in H$  implies the equation that follows, for that is what the sentence says up to end of the equation. Just reword it like this: "If  $f \in H$ , then the equation  $g(x) = \sum a_i f(x+i)$  defines ....". Another example, where the English sentence is very badly disrupted by some mathematical symbolism: "Let f belong to  $L^p(\mathbb{R})$  for some 1 ." The author could have repaired this by writing "Let <math>f belong to  $L^p(\mathbb{R})$  for some p satisfying 1 ." Don't say "Let

- $n \ge 1$  be an integer". What you mean is "Let n be an integer greater than or equal to 1." Or (better) "Let n be a positive integer." In all these cases, we are trying to make the sentence read correctly in simple English.
- 10. There is a difference among these entities: an element  $x_n$ , a countable set  $\{x_n : n \text{ is a natural number}\}$ , and a sequence  $[x_n : n \text{ is a natural number}]$ . To illustrate, let us note that  $[1,3,6,3,2,...] \neq [1,3,6,2,...]$ , but on the other hand for simple sets we do have  $\{1,3,6,3,2\} = \{1,3,6,2\}$ .
- 11. Quoting Halmos: "A sentence such as 'Whenever a positive number is  $\leq 3$ , its square is  $\leq 9$ ' is ugly." Ugly turns to confusing if you write "The degree of p=1". Say simply "The degree of p is 1. Don't write "The degree of p is  $\leq 3$ ."
- 12. Be very cautious about abbreviations. They should be avoided, in general. It is much better to say "Equation (7)" than "Eq. (7)". It is better to write "page 4" than "p.4". (There are exceptions. For example, some journals have specific style rules that go counter to the suggestions here.)
- 13. For the correct abbreviations of journals, consult the 32-page document "Abbreviations of Names of Serials Reviewed in Mathematical Reviews", published by the American Mathematical Society. This list is updated every year and appears in the Mathematical Reviews Annual Index.
- 14. Here is a tricky problem: What do you understand by the sentence "Let  $(x_n)$  be a sequence in  $\ell^2$ ." If you ponder this a little you will see that it is ambiguous, for every element of  $\ell^2$  is a sequence. So, does the author mean that  $x \in \ell^2$  or does he mean that  $x_n \in \ell^2$  for  $n = 1, 2, 3, \cdots$ ? Here we could refer to point 6 above. A sequence is a function on  $\mathbb{N}$ . So, if x is the name of a sequence, then  $x_n$  is by convention one of its values. We therefore should not say "consider the sequence  $x_n$ " if we really mean the sequence  $x = [x_1, x_2, \ldots]$ . J. L. Kelley, in "General Topology" advocated the notation  $(x_n, n \in \mathbb{N})$ . In the later book (Kelley, et al.) "Linear Topological Spaces" he prefers  $\{x_n, n \in \mathbb{N}\}$ . The latter is dangerous, for it is easily confused with  $\{x_n : n \in \mathbb{N}\}$ , which is quite different. (See item 10 above.) A suggestion: use  $[x_n]$  or  $[x_n, n \in \mathbb{N}]$ , or  $x : \mathbb{N} \to \mathbb{R}$ . Obviously the last one is best. It even displays the range of the sequence.
- 15. We wish to inveigh against using  $x^*$  as a generic element in a conjugate Banach space  $E^*$ . Usually, in functional analysis, the symbol \* is used to signify a certain close relationship to another entity, un-starred. For example, if E is the space,  $E^*$  is not just some other space: it is the conjugate of E. Likewise if A is an operator,  $A^*$  is not simply another operator: it is the adjoint of A. Using  $x^*$  as an entity bearing no relation to x goes counter to this style. A second reason for this advice is that some workers, when dealing with the space C(X), like to use  $x^*$  as the point-evaluation functional corresponding to the point x in X. In that context,  $x^*$  has this meaning:  $x^*(f) = f(x)$ , for all f in C(X). A corollary of all the previous discussion is that one should not write "Let  $E^*$  be an arbitrary Banach space." nor "Let  $L^*$  be an arbitrary linear transformation." nor "Let  $\hat{f}$  be an arbitrary continuous function on the domain X."
- 16. Don't say "This result is due to [7]". Say instead "This result was proved in [7]". We interpret the symbol [7] as standing for the reference in question, not for its author.
- 17. Don't say "These functions satisfy the following properties." Say instead "These functions possess the following properties." Don't say "These matrices have the following conditions." Say instead "These matrices satisfy the following conditions."
- 18. If you have written "In the case where...", ask yourself whether it may be simpler to write "If ...".
- 19. It is preferred to write multiple references in the style [5, 13, 23] instead of [5], [13], [23].
- 20. Write Equation (2.3), not equation (2.3) or just (2.3). Here, we want to make it easy on the reader, who may think that (2.3) refers to Section 2.3, not Equation (2.3). Notice that we capitalize "Equation", because we interpret "Equation (2.3)" as a proper noun, i.e., the name of something. Similar remarks apply to Theorem 8 (not theorem 8), and so on.
- 21. Try to avoid LONG theorems. Remember, the reader may be looking for the highlights of your work, and she will read any short theorem to see (a) whether it makes sense, (b) whether she can understand it, and (c) whether it is interesting. She probably will skip over any theorem more than four or five lines long if she is only trying to get some idea of what you are doing. Tricks for making theorems shorter: First, be sure that you are not burying definitions in the statements of your theorems. Second, consider transporting some of your hypotheses to the paragraph just before the theorem. This certainly is a good idea if you are stating some global hypotheses that apply to more than one result. Finally, consider breaking your theorem into several smaller digestible theorems. Do NOT attempt to shorten theorems by using abbreviations! The most memorable theorems are invariably the short ones that can be stated in ordinary words.
- 22. A theorem with a name (such as "Sampling Theorem") is likely to be more eye-catching than one without a name. Try to think of names that convey quickly what the theorem is about.
- 23. A function cannot be orthogonal or orthonormal; these are properties of sets of functions. Thus it is incorrect to say "The functions  $u_n$  are orthogonal". One should say, "The family  $\{u_n : n \in \mathbb{N}\}$  is orthogonal". Similar remarks pertain to the term "closed", in its many contexts. Do not say "The Borel functions are closed under addition." Say "The set of Borel functions is closed under addition." Similar remarks apply to the term "linearly independent". Again, it is

only sets that can be linearly independent; the term is not properly applied to elements of a linear space. Some may argue further that it is only indexed sets to which the term can be applied. Think of a matrix in which one row is a multiple of another. We would usually say that the set of rows is linearly dependent. But suppose that one row is a duplicate of another row. Then the set of rows may very well be linearly independent, for the SET is not changed when we remove the duplicate row! Thus, the phraseology "..... are linearly independent" is almost always wrong. Another similar situation: an integer cannot be relatively prime. Hence we should not speak of relatively prime integers. Pairs of integers can be relatively prime. So we should not say "Let n and m be relatively prime integers." Better: "Let m, n be a relatively prime pair of integers." Is it worthwhile to make these fine distinctions? Of course it is, especially in didactics, for we want to encourage students to think of properties of sets, when that concept is appropriate. Misusing the English will confuse students. Another instance where sloppy English is common but can be easily repaired: the phrase "....sets are disjoint". This usage insinuates that disjointness is a property that a set may have. But it is only pairs of sets or families of sets that can be disjoint. Thus, "a disjoint pair of sets" makes sense, as does "a disjoint family of sets". But "a family of disjoint sets" does not make sense. There is a standard definition: A family of sets is said to be disjoint if, for any two members A and B of the family, A and B have a nonempty intersection only if A = B.

- 24. Remember that you are writing for an international audience, for many of whom English is not a familiar language. In deference to them, do not use slang, colloquial English or informal figures of speech that may not be understood. Don't speak of "plugging" an expression into an equation. Don't say that a function "lives" on a set (when you mean that its support is contained in that set). Be sensitive to the needs of those who will be reading your work with a foreign-language dictionary at hand. In this connection, observe that most contractions ("isn't", "can't", "we'll", etc.) are colloquialisms suitable for speech but not for formal mathematical exposition. Don't say, "We'll now assume ...". Say instead, "We shall now assume...". The usage "We claim ..." is a bit colloquial. It is better to say "We assert....". Then, later, you can say, "This proves the assertion", which is much better than "This proves the claim". (Actually, "proving a claim" has a technical meaning in mining parlance!)
- 25. If your writing is good, the reader will form the subtle impression that you know what you are doing and that you are careful about details. He/she may even think that you have this attitude towards your mathematics as well as your writing! This will dispose her/him favorably toward your work. Instead of assuming that everything you have done is wrong unless proved otherwise, he/she may assume that everything you have done is correct unless proved otherwise. You don't want the reader to doubt the validity of your work in general simply because your writing has flaws! At some point, one or more readers of your work will be editors or referees. Try to impress them with the quality of your writing.
- 26. Clauses in a sentence must be separated with the semicolon (;). That is what the semicolon is for. For punctuation (and many other matters) you cannot go wrong by adopting as your reference the Merriam-Webster Collegiate Dictionary, for example the Tenth Edition. Its appendix on Style is 22 pages long and covers most questions likely to arise.
- 27. Avoid "dangling participles". These are participles (which act like adjectives) that have no noun to modify. Here is an example from a famous book: "Choosing q so that 1/q + 1/p = 1, Hölder's inequality shows...". Who is doing the choosing here? Surely not Hölder's inequality! One might say, "Choosing q so that ..., we find with the aid of Hölder's inequality that ..." Perhaps better, "If q is chosen so that ..., then Hölder's inequality shows that...".
- 28. There are many suggestions about the details of TEX that could be made here. For example, the symbol ^ is almost always too small. It is often used for the Fourier transform. It should be rendered in TEX by \widehat, and will come out looking like this: ^. A similar remark concerns the tilde, ~ , which is usually too small. Use instead \widetilde, which will appear as ~.
- 29. Don't say "If we are in a Hilbert space, then...". It is much too informal—almost slang.
- 30. In functional analysis, the term "fundamental" has been sanctioned for a long time in designating a set whose linear span is dense in the space. See Banach's book, page 58. Thus, the usage dates from 1932 if not earlier. The term "complete" should not be used for this concept. Here, too, we can rely on usage established by Banach. See pages 72 to 73. Of course, the word "complete" has a very special and universally accepted meaning when applied to metric spaces, and this alone should discourage its use with any different meaning.
- 31. Don't say "Using Theorem 7, we get that...". English has many better words than "get" to use in this context; for example, "infer", "conclude", "deduce" are all superior.
- 32. (Repeating ourselves a bit here!) Don't say "Let f(t) be measurable" when you mean "Let f be measurable". You can really confuse readers with this kind of writing, in addition to irritating them! If you have written  $f(t) \in L^2(\mathbb{R})$ , it is probably NOT what you mean; but it MIGHT be what you mean. That is why it is very confusing to use the wrong symbol. There is one well-known situation where making these distinctions is crucial: if f is a linear mapping, then its Fréchet derivative satisfies f'(x) = f; it does NOT satisfy f' = f, nor f'(x) = f(x).

- 33. Be especially careful in formulating definitions. Here is a BAD example, taken from an important book: We denote by  $\operatorname{pr}_E x$  the projection of x on E, and by  $\operatorname{pr}_E f$  the function defined by  $\operatorname{pr}_E f(x) = f(\operatorname{pr}_E x)$ . Here's another BAD example, also taken from a famous book: For a function f of two variables, define  $f_x(y) = f(x,y)$  and  $f_y(x) = f(x,y)$ . (!) It is embarrassing to have to remind mathematicians (of all people) that A DEFINITION MUST NOT BE USED TO ASSIGN TWO DIFFERENT MEANINGS TO THE SAME ENTITY OR SYMBOL.
- 34. In stating results formally in Theorems, Lemmas, etc., try to use English instead of symbols, and do not introduce symbols that are unnecessary for the statement of the result. In particular, do not introduce a symbol for the sole reason that you wish to use that symbol in the PROOF. Here is an example from a famous book: "If A is a Hermitian operator, then  $|A| = \alpha = \sup\{|\lambda| : \lambda \in \Lambda(A)\}$ ." (The  $\alpha$  is NOT needed in the theorem, but will be needed in the proof. It clutters the theorem unnecessarily.) Here are more examples from famous books: "A metric space X is compact if and only if it is both complete and totally bounded." (The X is not needed, and in fact is downright distracting.) "If A is a Hermitian operator, then  $\Lambda(A)$  is a subset of the real axis." (It would have been better to say, "The spectrum of a Hermitian operator is a subset of the real line".) "Let f be a continuous mapping of a compact metric space X into a metric space Y. Then f is uniformly continuous." (It would be better to say "A continuous mapping from a compact metric space into a metric space is uniformly continuous.") Here is another example from a famous book: "Let f(z) be a function analytic in the ring-shaped region between two concentric circles C and C', of radii R and R' (R' < R), and center a, and on the circles themselves. Then f(z) can be expanded in a series of positive and negative powers of z-a, convergent at all points of the ring-shaped region." It would be much neater to say ".... between two circles having center at a and on the circles themselves ....". The author introduced C, C', R, R' solely because he needed the names in the proof. But they certainly clutter the statement of the theorem. Of course, modern writers would refer to the function f, not to the function f(z).
- 35. In case the reader thirsts for more examples of bad writing, here are a few. (NOT made up by me!) "If f and  $g \in C[a,b]...$ ". "Let  $0 \le j \le m$  be a fixed integer." "If  $E \subset L_p(\mu)$  is n-dimensional,..."
- 36. Don't begin a sentence with a conjunction, such as "And", "Or", or "But". Try to avoid starting a sentence with "Then". (There are cases when it seems to be the better of two alternatives.)
- 37. In defining sets, there are two notations in common use, viz.,  $\{\cdots \mid \cdots\}$  and  $\{\cdots : \cdots\}$  The latter is to be preferred because the symbol ":" is not often used in any other technical context, whereas the symbol "|" is used for absolute values, for norms and for restriction operators. (The principal other use of ":" seems to be in defining a map, as in  $f:\Omega \to \mathbb{R}$ .) Thus, we can use ":" and thereby avoid such monstrosities as this:  $\{|r| \mid |r+1| < 1\}$ . It will look much better as  $\{|r| : |r+1| < 1\}$ . Maybe some adventurous mathematicians can introduce a special symbol for this important construct; how about :? (It's already available in  $T_EX$ .)
- 38. Due principally to the malevolent influence of ill-educated journalists, there is a current tendency in writing to abandon certain forms and styles that were serving important purposes in the past. These forms should be retained, even in the face of copy editors' objections. Here are some examples. At one time, foreign words and phrases were set in italics as a matter of style. This facilitated the reading of the text, as it warned the reader that something outside of normal English was involved. Here is an illustration of the old style (the style that we think should be retained): "The theorem will be proved under minimal a priori assumptions on the function." The italicization warns the reader that "a" is not the English article but a Latin preposition; also if the reader is reading out loud, it warns him about the pronunciation. Another example: "The proof is the same, mutatis mutandis."
- 39. Another style question arises in the spelling of words like "cooperate", "zoology", "coordinate", "reinforce". The old style would write "coöperate", "zoölogy", "coördinate", "reënforce". This device helps the reader in pronunciation, if nothing else. We also note that this style is embraced by the *New Yorker* magazine, which has for many years been a paragon for style.
- 39. In lectures one often hears the tilde ( ~ ) referred to as "a twiddle". This should be regarded as a sort of baby talk—permissible (because it is faintly humorous) in a lecture, but unacceptable in written text. Similar remarks apply to the circumflex ( ^), often called "hat". There are university professors on record who didn't know the Greek alphabet, and therefore invented their own childish names, such as "bird" for γ.
- 40. In formal writing, the end of a proof should be signaled somehow. The symbol was introduced by Halmos, and has proved its value. It certainly is better than some loose slang such as "We're done." If you do not like the black box, try "This concludes the proof".
- 41. If you are proving a theorem a second time, do not say, "We will now reprove Theorem 8." If the reason for this is mysterious to you, consult a dictionary for the meaning of "reprove".

- 42. Be very careful about saying that one theorem is equivalent to another. This is nonsense, because all theorems (being true) are equivalent to one another. Usually such a statement is intended to convey the idea that each of the two theorems follows EASILY from the other. In other words, it is a statement about possible PROOFS, not about the theorems themselves.
- 43. Don't sacrifice clarity to brevity. Avoid such writing as "A real valued function of a real variable, defined on some neighborhood of 0, is said to be o(t) if  $\lim_{t\to 0} o(t)/t = 0$ ". (This is taken from a famous book.)
- 44. If you are using material taken from another source, it is essential that this fact be acknowledged and that the source be fully identified. Not observing this rule opens the writer to an accusation of plagiarism. So, be sure to credit your sources, even if you are not using a direct quote. Short quotations are permitted by the copyright laws; long ones are not. These matters are governed not only by copyright laws but also by scholarly standards and traditions.
- 45. Mathematicians sometimes take a long way of saying things. For example: "The complex number z is a limit of zeros of  $S_n$  if and only if  $z \in \{z \in \mathbb{C} : \cdots \}$ ." One could have said instead "The complex number z is a limit of zeros of  $S_n$  if and only if  $\cdots$ ."
- 46. Here is an example where item 6, above, was ignored:  $||f(x) e^x g(x)||$ . How can it be repaired? Perhaps this is best:  $||f g| \exp||$ .
- 47. Ignoring item 7, above, leads to this: "Let G be an open bounded set in  $\mathbb{C}$  which can be represented...".
- 48. Surely this is weird: "The constant c depends only on  $\alpha, \beta, g, q, k$ , and  $\delta$ ."
- 49. Confusing bound and free variables leads to this nonsense (an actual quote): "If L is a linear functional,  $L^xF(x,y)$  means that L is applied to F as a function of x, and  $L^yF(x,y)$  means that L is applied to F as a function of y." We can only wonder what  $L^xF(y,x)$  means. There is a standard remedy for this difficulty: given a function f of two variables, define its "sections",  $f_x$  and  $f^y$  by the equations  $f_x(y) = f(x,y)$  and  $f^y(x) = f(x,y)$ . Then if L is a functional, it makes good sense to write  $Lf_x$  or  $Lf^y$ . The meaning of  $f_a(w)$  is unambiguous: it is f(a,w). Likewise  $f^{\Omega}(\Delta)$  is  $f(\Delta,\Omega)$ . See item 33, above, for similar confusing definitions, and for the "axiom" that says you should not define something to mean two different things. (Echos of "Alice in Wonderland".)
- 50. In general, do not depart from standard ways of doing things. You'll run into problems with journal editors if you do. For example, take the trouble to put bibliographies in alphabetical order. That's what most readers and editors expect or insist on. Use standard abbreviations of Mathematical journals. (Item 13 above.)
- 51. Another example where principles above have been violated: Theorem. Let f'(x) be continuous and invertible at  $x = x_0$ , and suppose  $|f'(x_0)^{-1}| < \eta$ . If  $|f(x_0)|$  is sufficiently small, Newton's method converges to a solution of f(x) = 0.
- 52. Avoid awkward neologisms. They are difficult for any foreign reader who might be using an old dictionary. Besides, many of these are jargon. In this category are "prioritize", "re-mediation", "importantly". Many such words are invented by business people, politicians, and others whom we should not imitate, except in jest.
- 53. Here is a suggestion related to items 21 and 34 above. Try to make the statement of theorems as simple and clear as possible. Avoid jargon and notation as much as possible. In particular, don't use the statement of a theorem as a place to insert a definition. Here is an example where this axiom is violated: THEOREM. A point  $\zeta$  is an accumulation point of the zeros of  $\{f_n(z)\}$  if and only if  $\zeta \in D_{\infty}$ , where

$$D_{\infty} = \{ z \in \mathbb{C} : bla \ bla \ bla \}$$

54. And now for some tongue-in-cheek admonitions: Never use the word "majority" without the qualifying adjective "vast". Probably in a few years this advice will not be necessary, because we will all be writing it as one word: "vastmajority". In the same vein, let me say that you are free to use the word "decimate" in any sense whatsoever. (Constant misuse over the years has deprived the word of any definite meaning.) Here is a good place to advise that if one does not know the meaning of a word, he/she has every right to consult a dictionary! For some words, such as "decimate" or "reticent", it may be prudent to do this in order to learn the latest meaning. (A vast majority do not use these two words correctly. Other words in this category could be cited.) Remember this sample as a model of correct usage: "The boxer was reticent about entering the ring because he knew that Killer McCoy would decimate him." Be sure, when you use the word "unique" that you precede it with a suitable adverb, such as "very" or "most" or "awfully". And remember, to be current with the best writing style, you must read your daily newspaper and one of the style-setting magazines such as "Newsweek" or "Time". In doing so, you will learn never to use the word "growth" wihout the obligatory adverb "exponential". This is especially important for a mathematician, of course.

# Selected References

- 1. Gillman, L., "Writing Mathematics Well", Mathematical Association of America, 1987.
- 2. Steenrod, N. et al., "How to Write Mathematics", American Mathematical Society, 1973. (In this collection there is a noteworthy essay by Paul Halmos that mathematical writers should read and take to heart.)
- 3. Knuth, D., "The TeXbook", Addison-Wesley Publishing Co., London, 1984.
- 4. Lamport, L., "LaTeXUser's Guide and Reference Manual", Addison-Wesley Publishing Co., London, 1986.
- 5. Knuth, D., T. Larrabee, and P.M. Roberts, "Mathematical Writing", MAA Notes, No. 14, Math. Assoc. of America, 1989
- 6. Sawyer, S.A., and S.G.Krantz, "A TEX Primer for Scientists", CRC Press, Boca Raton, Florida, 1995.
- 7. Hwang, A.D., "Writing in the age of LaTeX", Notices of the AMS, 42 (1995), 878-882.
- 8. Fowler, H.W., "Modern English Usage", Cambridge University Press, 1926. (New edition in 1997!!!.)

#### Some Choice Quotes

- 1. "In 1940 I wrote a thesis, Whyburn made me revise it, McShane made me revise it again, and Hedlund said he'd revise except it was too late in the year. So it was accepted and then Sammy Eilenberg spent a couple of weeks revising and making me revise. This training, with a post-doctoral bit from Paul Halmos a few years later, is how I learned to write mathematics." (from "Once Over Lightly", by J. L. Kelley, in "A Century of American Mathematics, Part III", Peter Duren, ed., Amer. Math. Soc. 1989.)
- 2. "A doctoral candidate under his supervision could always expect to prepare at least twenty drafts of his dissertation before its linguistic format would be approved". (A comment about Professor Walter B. Ford at the University of Michigan, quoted from his obituary, Amer. Math. Monthly 78 (1971), 1094-1097.)

## Some Writers to Emulate

Paul Halmos, L. Gillman, I.J. Schoenberg, J.L. Kelley, Walter Rudin, W. Hurewicz,...

## Some Writers NOT to Emulate

Well, we have to avoid libel, don't we?